Coupled Pendulums

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**Objective:**

To study the oscillations of a string under an external driving force, as well as to observe experimentally the effects of a change in frequency on the amplitude.

**Description:**

A rod was fixed at one end of the table and on end of the cord was attached to it. The pulley was attached to the other end of the table and the end of the cord with the mass attached to it was passed over the pulley. A mechanical wave driver was placed on the table such that the cord either passed through it, or close enough to it for the wave-driver to induce oscillations of the cord. Special care was taken to ensure that the cord did not bend due to the mechanical wave driver.

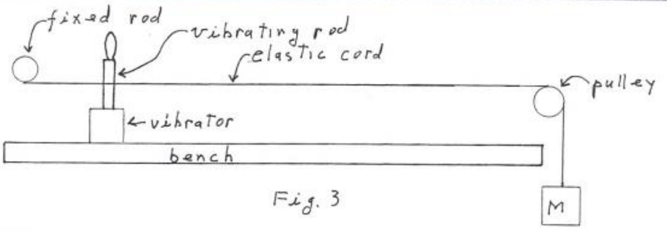


Fig. 3

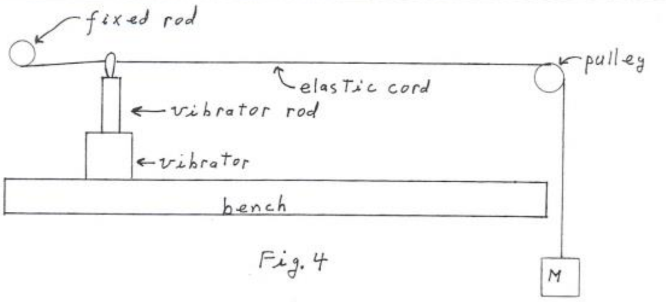


Fig. 4

**Theory:**

For the initial part of the experiment, we were required to place the mechanical wave driver so that the cord came in contact with it but was not passed through it. In this case, we assumed the vibrations occurred in the x-y plane alone (with the length of the string along the x axis). Since the cord was fixed at both ends, we assumed the process was analogous to a standing wave. Given a tension T and a mass per unit length of $\rho$ (and assuming there was no damping), we arrived at the following equation:

$$\frac{\partial^2 y}{\partial x^2} - \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2} = 0$$

The most general solution to this equation is g(x$\pm$vt) where g is any function and v = $\sqrt{(T/\rho)}$

This solution represents a wave travelling along the x axis with a speed of v. the boundary conditions of the experiment provide us with the equations required to solve for this setup. We can see that the oscillations resemble a sinusoidal wave and the boundary conditions limit the possibilities for the frequencies (to discrete values). Using this, we get the following equation for the normal modes:

$$y(x,t) = f(x)\cos\omega t$$

Here $\omega$ is the angular frequency and $t$ the time. Substituting this in our earlier equation we get:

$$\frac{d^2f}{dx^2} + \frac{\rho}{T}\omega^2 f = 0$$

Solving this for our boundary conditions gives us:

$$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3 \ldots.$$

From this we can derive that the linear frequency is:

$$\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\rho}}$$

For a given sinusoidal wave the wave relationship $\lambda\nu = v$ still applies. If g(x±vt) is substituted in place of f, we get:

$$velocity = \sqrt{\frac{T}{\rho}}$$

This tells us that for a given string at a fixed value of T, the velocity of the oscillations is independent of the frequencies or the wavelengths of the oscillations.

For the second part of this experiment we used strongly driven oscillations with the same boundary conditions as before. In this case the frequency of the oscillations is the same as the frequency supplied by the wave driver i.e. it is chosen by us. Assuming that f(x) was of the from $A \sin\left(\frac{2\pi}{\lambda}x\right)$. From this we get the equation:

$$y(L,t) = A \sin\left(\frac{2\pi}{\lambda}\right) L \cos \omega t = B \cos \omega t$$

Here B is termed the driving amplitude and is determined by the experimenter. The unknown, A can then be found using:

$$A = \frac{B}{\sin\left(\frac{2\pi}{\lambda}L\right)}$$

Which means the amplitude of the wave reaches its minimum when the sine is at $\pm 1$

The minimum response of the string in this case is obtained when the driver is at the point of maximum displacement i.e. the point where the cord passes through the mechanical wave driver is the anti-node of the wave. In this case we have:

$$\lambda_n^{min} = \frac{4L}{n} \qquad \text{and} \qquad \nu_n^{min} = \frac{n}{4L}\sqrt{\frac{T}{\rho}} \qquad where\ n = 1, 3, 5\ ....$$

The maximum response is obtained when the value of the sine is 0 i.e. when the driving force acts at a node. This is evidently a contradiction in itself because if a point is being driven, it cannot be a node. It is however reasonable to assume that the maximum amplitude will be obtained at frequencies close to those derived from this assumption i.e.:

$$\lambda_n^{max} \cong \frac{2L}{n} \qquad \text{and} \qquad \nu_n^{max} \cong \frac{n}{2L}\sqrt{\frac{T}{\rho}} \qquad where\ n = 1, 2, 3\ ....$$

This predicts that the change in string response with respect to the frequency will be a continuously changing function.

**Data and Analysis:**

Length of cord = 1.043m

Mass of cord = 6g = $6\times 10^{-3}$kg

Linear Mass density of the cord = $5.75 \times 10^{-3}$kg/m

**4.2 Frequency (of the second mode) against Tension**

| | Mass ($\times 10^{-3}$kg) | Tension (N) | Experimental Frequency (Hz) | Δ Length ($\times 10^{-2}$m) | Expected Frequency (Hz) |
|---|---|---|---|---|---|
| **1.** | 100 | 0.98 | 18 | 1.2 | 13.9 |
| **2.** | 200 | 1.96 | 26 | 8.5 | 24.6 |
| **3.** | 400 | 3.92 | 43 | 74.5 | 40.1 |

The experimental frequencies come within an acceptable range of uncertainty of the expected frequencies.

**4.3 Mode Frequencies**

Mass = 200g

| Mode | Nodes | Frequency (Hz) |
|---|---|---|
| **1** | 0 | 13 |
| **2** | 1 | 26 |
| **3** | 2 | 39 |
| **4** | 3 | 51 |

These frequencies demonstrate a linear relationship to the mode as would be expected from our initial equation i.e. multiples of the first mode.

**4.4 Between Modes (between 13 Hz and 26 Hz)**

| | Frequency (Hz) | Amplitude ($\times 10^{-2}$m) |
|---|---|---|
| **1** | 13 | 0.75 |
| **2** | 16 | 0.50 |
| **3** | 21 | 0.15 |
| **4** | 25 | 0.25 |
| **5** | 26 | 0.50 |

**5 Wave Latency**

On setting the first string vibrating in its 1st mode and stopping the wave driver and then restarting the wave driver, the string took 1.35 seconds for the oscillations to build up. This could be due to the damping of the oscillations

**7.1**

The distance from the wave driver to the pulley was: 0.879m

**7.2 Resonances**

Mass = 200g

| Mode | Nodes | Frequency (Hz) | Expected Frequency (Hz) |
|---|---|---|---|
| **1** | 0 | 13 | 12.5 |
| **2** | 1 | 26 | 24.9 |
| **3** | 2 | 39 | 37.4 |
| **4** | 3 | 51 | 49.8 |

This demonstrates the linear relationship of the frequency to the mode number as would be expected from our equation. Resonances do resemble the normal mode vibrations. The driving point should be negligibly close to the node. Since we see that the driving point moves significantly less than the maximum amplitude, and we do not see a node between the driving point and the first anti-node the node must be on the side of the driver away closer to the fixed rod.

**7.3 Anti-Resonances**

The three lowest anti-resonances were found each between two consecutive modes

| Mode 1 | Mode 2 | Frequency (Hz) | Frequency (Hz) |
|---|---|---|---|
| **1** | **2** | 18 | 18.7 |
| **2** | **3** | 30 | 31.2 |
| **3** | **4** | 47 | 43.6 |

Since the amplitudes of the anti-resonances are so small, it is impractical to detect the actual position of the anti-node. However, the driving point should be at the anti-node as proven by our theoretical hypothesis and corroborated by the observation that no point on the cord seems to move more than the driving point.

**7.4 Between Modes (13 Hz and 16 Hz)**

| | Frequency (Hz) | Amplitude ($\times 10^{-2}$m) |
|---|---|---|
| **1** | 13 | 1.80 |
| **2** | 15 | 0.25 |
| **3** | 19 | 0.10 |
| **4** | 23 | 0.20 |
| **5** | 26 | 1.20 |

We notice that these results represent a pattern resembling the observations in part 4.4 but with larger amplitudes. The larger amplitude in this part of the experiment is because the oscillations are (comparatively) free in the first part and therefore have a smaller force acting on them.

**Original Experiment:**

For the original part of the experiment, we used the second normal mode of the cord as a control setup and added varying masses at the nodes to observe their effects

| | Mass ($\times 10^{-3}$kg) | Amplitude ($\times 10^{-2}$m) |
|---|---|---|
| **1** | 0 | 1.00 |
| **2** | 20 | 1.20 |
| **3** | 50 | 1.40 |
| **4** | 100 | 1.70 |

From this we observed that the mass acts like a reflecting surface for the oscillations/energy i.e., when the mass was placed at a node, the amplitude on the side of the mass away from the wave driver decreased and the amplitude closer to the wave driver increased which would be consistent with the effects of superposition of wave oscillations. The effect becomes increasingly visible as the mass is increased and the amplitude increases even more. This also demonstrates conservation of momentum where, to move a larger mass by the same (negligibly small) distance, a much larger change in momentum is required. This change in momentum shows up as the hypothetical wave that is superimposed on the initial wave to create a standing wave of larger amplitude.

**Error Analysis:**

A large part of the uncertainties that we see in this experiment can be attributed to the fact that we do not correct for damping. Additionally, the amplitudes in many parts of the experiment small enough to cause significant uncertainties in the final values.